For the very small diameter filaments considered, the radiation Nusselt number is a negligible correction to the convection Nusselt number, as shown in Fig. 1.

Apparently then, for materials that have very high thermal diffusivities as compared with that of the cooling medium, the assumption of isothermal cooling as described by Bourne and Elliston [1] is satisfactory for the prediction of heat transfer from continuous surfaces. However, all necessary energy losses, such as radiation, must be included in the calculations, for, as is demonstrated in this paper, these corrections can be quite significant.

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A GAS VOLUMETRIC TECHNIQUE FOR THE MEASUREMENT OF HEAT TRANSFER COEFFICIENT IN POOL BOILING

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NOMENCLATURE

- h, boiling heat transfer coefficient;
- K, thermal conductivity;
- z, length;
- N, air moles;
- P, pressure;
- R, Avogadro constant;
- r, radial coordinate;
- R, radius;
- T, temperature;
- T_{CW}, temperature of cooling water inlet;
- T_{BW}, temperature of water boiling outside the evaporator;
- T_{PE}, temperature of outer wall of the evaporator;
- T_P, temperature of the inner wall of the evaporator;
- V, volume;
- W, power supply [W];
- W_{diss}, power dissipated in the copper electrode;
- z, axial coordinate.

Greek letters

- α, temperature coefficient of thermal conductivity.

Subscripts

- 1, radius of the copper electrode;
- 2, inner radius of the evaporator;
- 3, outer radius of the evaporator;

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B,	volume occupied by air in the measurement burette;
C,	volume occupied by air in the evaporator;
I,	section I,
II,	section II.

Superscripts

$^{(0)}$,	standard condition;
1,	measurement condition.

DESCRIPTION OF APPARATUS

A TECHNIQUE is presented herebelow to measure heat transfer coefficients in pool boiling on vertical cylindrical evaporators. The temperature of the outer wall, on which boiling takes place, is measured by means of a gas volumetric method. The experimental equipment is sketched in Fig. 1: evaporator A, completely immersed in water, consists of a 304 SS tube 50 cm long (20 mm i.d., 21 mm o.d.), welded to flange F, while its closed plane top is brazed to a copper electrode E. Low voltage a.c. power (max 10 kW) is applied between the flange and the copper electrode, so that heat is generated through the Joule effect.

The electrical resistance of the circuit is such that heat generation is mostly concentrated in the thin walls of the evaporator itself. On the other hand while the evaporator is cooled by external boiling water, the inner copper electrode is not able to dissipate the heat generated by electric current flowing inside, without reaching an excessive temperature. Therefore it is cooled by water in forced convection along a certain part of its length: water enters in the inner duct I, flows upward, inverts flow direction at the top and comes out at C. Section II of the electrode is made of solid copper. The annular space S between the copper electrode and the inner surface of the evaporator wall is sealed up airtight, except for a connection to a Schiff semimicro-azotometer by means of a metal capillary tube (0·6 mm i.d.), as shown in Fig. 1.

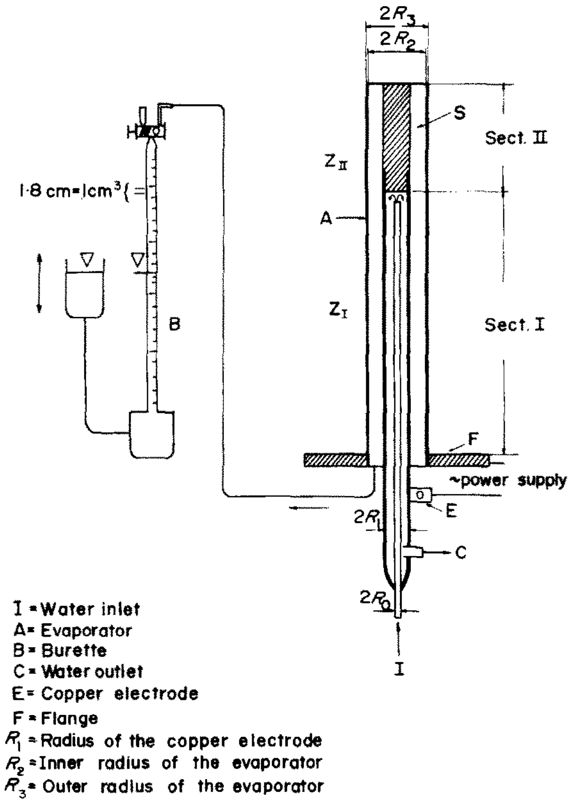

FIG. 1.

MEASUREMENT METHOD

The changes in volume of the air trapped inside the cavity S in a reference condition are measured for an indirect determination of the outer wall temperature of the evaporator when heated by the power supply. The changes in volume are evaluated by reading the variations of the liquid level in the azotometer after having again established the atmospheric pressure. This method was preferred to that of the use of thermocouples for two main reasons:

(a) it is very difficult to locate thermocouples in the narrow space available in this particular arrangement. Moreover, the internal copper electrode being cooled, thermocouples are influenced by its temperature and no efficient thermal screening is feasible;

(b) measurement of the wall temperature by a gas volumetric method already provides average values, while to obtain the same results with thermocouples it would be necessary to use a lot of them.

PHYSICAL ASSUMPTIONS

A reference condition for the gas thermometer is defined as follows: (a) no power is supplied; (b) the temperature of the cooling water along the first part of the inner electrode (length I) is constant, while in section II the temperature distribution is dominated by thermal conductivity. A measurement condition is defined as follows: (a) power is supplied; (b) an increase (function of the length) of the temperature of the external surface of the electrode takes place, because of power generation inside copper; (c) the cylindrical walls of the evaporator are superheated in comparison to that of boiling water (both for the temperature drop required to transfer heat from the outer wall to the boiling liquid and for the temperature drop across the thin metal wall).

As a general rule, the reference temperature of the water which is external to the evaporator was selected equal to the boiling water temperature in measurement conditions.

MATHEMATICAL PROCEDURE

The number of moles of air trapped in the cavity S, in the capillary tube and in the azotometer is constant, both in reference and in measurement conditions, that is

$$N_B^{(0)} + N_c^{(0)} = N_B^{1} + N_c^{(1)} \tag{1}$$

Assuming that the air has the properties of a perfect gas, we have

$$N_B + N_C = \int_{V_B + V_C} \frac{P}{RT}\,\mathrm{d}v. \tag{2}$$

The pressure is kept constant in all the system $V_B + V_C$ and the temperature field in the azotometer is also constant and equal to the room temperature T_B. The extremely small volume of the capillary tube is negligible. From equations (1) and (2) we find

$$\frac{V_B^{(1)} - V_B^{(0)}}{T_B} \equiv \frac{\Delta V_B}{T_B} = \int_{V_C} \frac{\mathrm{d}V_C^{(0)}}{T_C^{(0)}} - \int_{V_C} \frac{\mathrm{d}V_C^{(1)}}{T_C^{(1)}}. \tag{3}$$

By reading ΔV_B, one gets to know the right side of the equation (2). Due to the cylindrical symmetry of the system, $\mathrm{d}V = 2\pi r\mathrm{d}r\mathrm{d}z$ while $T = T(r, z)$ is the temperature of air trapped inside the cavity, function of radial and axial location. The size of the cavity S being so small, natural convection caused by internal temperature differences is not permitted (the critical Grashof number is much above actual values). Moreover axial conduction along the evaporator and the water cooled section of the copper electrode has negligible effects on distorsion of the thermal field. As a general rule one can therefore express the integrals in (2) as:

$$\int_V \frac{\mathrm{d}V}{T} = \int_0^L \mathrm{d}z \int_{R_1}^{R_2} \frac{2\pi r\mathrm{d}r}{T(r,z)}. \tag{4}$$

The radial distribution of temperature in S is calculated by means of the conduction equations

$$\mathrm{div}\, K\, \mathrm{grad}\, T = 0 \quad K = K_0(1 + \alpha T). \tag{5}$$

In reference conditions the boundary conditions are the following:

$$\text{for} \quad r = R_1 \quad T^{(0)} = T_{CW} \tag{6a}$$
$$r = R_2 \quad T^{(0)} = T_{BW} \quad \text{in Section I}$$

$$\text{for} \quad r = R_1 \quad T^{(0)} = T_{CW} + \frac{T_{BW} - T_{CW}}{Z_{II}} \tag{6b}$$
$$r = R_2 \quad T^{(0)} = T_{BW}$$

in Section II.

Since all the quantities in (5) are known, the first integral of the right side of (2) can be calculated. In measurement conditions, one has:

$$\text{for} \quad r = R_1 \quad T^{(1)} = T_{CW} + (a_1 + a_2 z + a_3 z^2)\, W \tag{7a}$$
$$r = R_2 \quad T^{(1)} = T_P \quad \text{in Section I}$$

$$\text{for} \quad r = R_1 \quad T^{(1)} = T_{CW} + (a_4 + a_5\,[T_P - T_{CW}]z + a_6)\, W \tag{7b}$$
$$r = R_2 \quad T^{(1)} = T_P$$

in Section II.

In the boundary conditions, a_1, a_2, a_3, ..., are numerical constants which depend on the values of the power dissipated in the inner electrode, and on the rise in temperature of the cooling water. In section I these quantities depend on the overall heat transfer coefficient of the inner heat exchanger and on the temperature rise by Joule effect of the wall of the inner heat exchanger.

In section II the temperature of the surface at radius r_1 depends on the heat generated by Joule effect and on the water temperature in the heat exchanger.

Moreover all other quantities in (6) are known, except T_P. Therefore, equation (2) can be solved analytically or numerically, for T_P. The temperature T_P (which is equal to the temperature T_{PE} in test reference conditions) is related to T_B, when power is supplied, by

$$T_{PE} = T_B - a_r R_3\left(\frac{1}{2} - \frac{R_3^2}{R_3^2 - R_2^2}\ln\frac{R_3}{R_2}\right) \tag{8}$$

where a_7 is yet another constant, which depends on the geometry and physical characteristics of the evaporator, and of the power dissipated in the external wall of the evaporator.

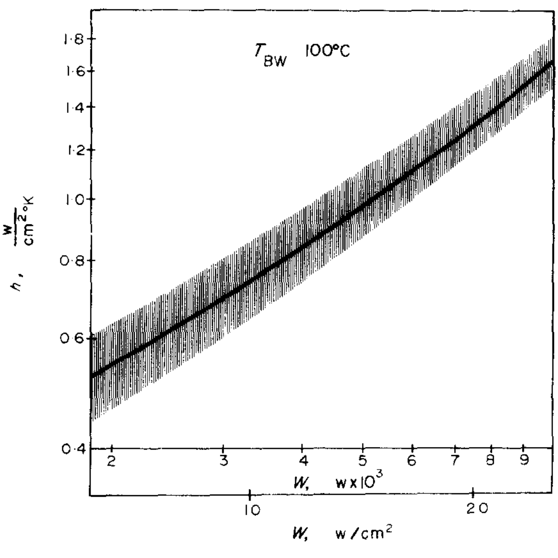

FIG. 2.

In fact, the mathematical model, due to its complexity, was solved by a UNIVAC 1180 computer. Some results are presented in Fig. 2.

CONCLUSIONS

In our experimental apparatus, the overall sensitivity is 0·75 $cm^3/°K$ at boiling water temperature of 100°C (the displacement corresponding to 1 cm^3 is 1·8 cm), while the intrinsic inaccuracy, shown by small oscillations of the liquid level of the azotometer, is in the order of 0·05 cm^3. Thus the procedure seems perfectly adequate for an accurate measurement of the heat transfer coefficient in pool boiling, given by:

$$h_B = \frac{W - W_{diss}}{2\pi R_3 L(T_{PE} - T_{BW})} \tag{9}$$

even if $(T_{PE} - T_{BW})$ is a few degrees. On the contrary the intrinsic uncertainty is caused by the irreproducibility of the boiling phenomenon itself.

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LIMITING NUSSELT NUMBERS IN FINITE MHD DUCTS

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NOMENCLATURE

B, magnetic flux density;
C_p, heat capacity;
D_e, equivalent diameter $[4a/(1 + 1/\gamma)]$;
E, electric field;
Ec, Eckert number $[\bar{u}^2/C_p(T_c - T_w)]$;
H^*, dimensionless magnetic field,
$$\left[H_x/a^2\left(-\frac{\partial P}{\partial x}\right)\left(\frac{\sigma_c}{M_c}\right)^{\frac{1}{2}}\right];$$
J, current density;
k, thermal conductivity;
M, Hartmann number $[aB_0(\sigma_c/M_c)^{\frac{1}{2}}]$;
N_E, electric field parameter $[E_0/\bar{u}B_0]$;
Nu, Nusselt number $[q_w D_e/k(\bar{T} - T_w)]$;
P, pressure;
q, heat flux;
u^*, dimensionless velocity, $[\mu_c u/a^2(-\partial P/\partial x)]$;
y^*, Y/z;
Z^*, Z/b.

Greek symbols
γ, aspect ratio of duct (b/a);
θ, dimensionless temperature, $T - T_w/T_c - T_w$;
μ, viscosity;
ρ, density;
σ, electrical conductivity.

Subscripts
c, centerpoint of duct;
w, wall.

Superscripts
′, "pseudo" parameter defined on basis of a velocity other than the average velocity;
—, average value;
*, dimensionless variable.

1. INTRODUCTION

THERE have been many analyses of MHD heat transfer in parallel plate geometries [1–5] but studies in finite ducts have been relatively scarce. Despite the abundant work on the simpler parallel plate problems, there has been surprisingly little emphasis placed on predictions of the Nusselt number. Finite ducts have not received as much attention due in part to the complexity of the problem since the fluid flow is influenced by the nature of recirculating currents which must be accounted for by relating the local current density to the magnetic field which is induced in the direction of the flow. This is particularly important when the duct walls are electrical insulators and it then becomes necessary to solve two coupled partial differential equations: the momentum equation and the equation describing the distribution of the induced magnetic field.